

# Back Reaction and Graceful Exit in String Inflationary Cosmology

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## Abstract

Classical string cosmology consists of two branches related to each other by scale-factor duality: a super-inflation branch and a Friedmann-Robertson-Walker (FRW) branch. Curvature and string coupling singularity separates the two branches, hence posing ‘graceful exit problem’ to super-inflationary string cosmology. In an exactly soluble two-dimensional compactification model it is shown that quantum back reaction retards curvature and string coupling growth and connects the super-inflation branch to the FRW branch without encountering a singularity. This may offer an attractive solution to the ‘graceful exit problem’ in string inflationary cosmology.

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In the Standard Model of big bang cosmology horizon and flatness problems constitute two notorious naturalness problems. Inflationary cosmology [1] was invented to solve these problems through a long period of accelerated expansion  $\dot{a}(t) > 0$ ,  $\ddot{a}(t) > 0$ . Three different types of inflation satisfy these conditions: de Sitter  $a(t) \sim e^{Ht}$ , power-law  $a(t) \sim t^p (p > 1)$  and super-inflation  $a(t) \sim (-t)^p (p < 0)$ . Among them super-inflation is regarded as the most attractive: super-inflation is driven by kinetic energy, hence, largely model-independent and free from fine-tuning problem typically present in potential energy driven de Sitter or power-law inflation models.

Veneziano [2] has first pointed out that, in classical string cosmology, thanks to the scale-factory duality the super-inflation arises naturally in addition to decelerating expansion of Friedmann-Robertson-Walker (FRW) type. One might then hope that superstring theory gives rise to a successful big bang cosmology starting from a super-inflationary branch and then smoothly evolve into a FRW-type branch. It has been noted, however, that such evolution cannot be realized since the super-inflation phase ends with divergent curvature and string coupling and cannot be continued to regular FRW phase. Because of this string inflationary cosmology is afflicted by a ‘graceful exit problem’ [3].

The fact that both curvature and string coupling grow arbitrarily strong as the super-inflation phase evolves is a universal feature of Veneziano’s string cosmology in all spacetime dimensions [4]. This indicates that classical description of string cosmology breaks down and that quantum effects modify evolution of the two classical branches significantly. One may then wonder if quantum back reaction can wash out curvature and string coupling divergences completely that a transition to FRW phase becomes possible. In this Letter, we show that branch change, hence, graceful exit is indeed possible after quantum back reaction is taken into account.

To address quantum back reaction in a controlled approximation we consider an exactly soluble two dimensional string compactification model. The model is described by the same action as the dilaton gravity of Callan, Harvey, Giddings and Strominger (CGHS) [5]

$$S_0 = \int \frac{d^2x}{2\pi} \sqrt{-g} [e^{-2\phi} (R + 4(\nabla\phi)^2 - 4\Lambda) - \frac{1}{2}(\nabla\vec{f})^2] \quad (1)$$

where  $\phi, \Lambda(\geq 0)$  and  $\vec{f}$  denote dilaton, cosmological constant (central charge deficit) and free  $N$ -component Ramond-Ramond scalar field. We first show that counterparts of Veneziano's two branches with accelerating and decelerating expansions are also present in two dimensions. Equations of motion derived from Eq.(1) are

$$\begin{aligned} e^{-2\phi} [4\nabla_\mu \nabla_\nu \phi + g_{\mu\nu} R] &= \nabla_\mu \vec{f} \cdot \nabla_\nu \vec{f} - \frac{1}{2} g_{\mu\nu} (\nabla \vec{f})^2; \\ R &= 4(\nabla\phi)^2 - 4\nabla^2\phi + 4\Lambda, \\ \nabla^2 \vec{f} &= 0. \end{aligned} \quad (2)$$

In conformal gauge

$$ds^2 = -e^{2\rho} dx_+ dx_-; \quad x_\pm = t \pm x, \quad \partial_\pm = \frac{1}{2}(\cdot \pm \imath), \quad (3)$$

the equations of motion Eq.(2) can be derived from the following gauge fixed action

$$\begin{aligned} S_0 = \int \frac{d^2x}{2\pi} [e^{-2\phi} (4\partial_+ \partial_- \rho - 8\partial_+ \phi \partial_- \phi - 2\Lambda e^{2\rho}) \\ + \partial_+ \vec{f} \cdot \partial_- \vec{f}] \end{aligned} \quad (4)$$

supplemented with constraint equations

$$T_{\pm\pm} = \frac{1}{2}(\partial_\pm \vec{f})^2 + e^{-2\phi}(4\partial_\pm \rho \partial_\pm \phi - 2\partial_\pm^2 \phi) = 0. \quad (5)$$

That the model is classically exactly soluble is seen by changing the field variables to [6,7]

$$\Phi = e^{-2\phi}, \quad \Sigma = 2\kappa(\phi - \rho) \quad (6)$$

where  $\kappa$  is a parameter introduced to keep track of quantum loop expansions. Then Eqs.(4, 5) become [6]

$$\begin{aligned} S = \int \frac{d^2x}{\pi} [ \frac{1}{2\kappa} (\partial_+ \Phi \partial_- \Sigma + \partial_- \Phi \partial_+ \Sigma) - \Lambda e^{-\Sigma/\kappa} \\ + \frac{1}{2} \partial_+ \vec{f} \cdot \partial_- \vec{f} ] \end{aligned} \quad (7)$$

and

$$T_{\pm\pm} = \frac{1}{2}(\partial_{\pm}\vec{f})^2 + \partial_{\pm}^2\Phi + \frac{1}{\kappa}\partial_{\pm}\Phi\partial_{\pm}\Sigma = 0 \quad (8)$$

respectively. In the classical limit  $\kappa \rightarrow 0$ , the theory has a useful  $\Sigma$ -field symmetry [7]

$$\Phi \rightarrow \Phi + \epsilon_{\Sigma}, \quad \Sigma \rightarrow \Sigma. \quad (9)$$

In what follows we restrict to the case of vanishing cosmological constant  $\Lambda = 0$ . In this case there is an additional  $\Phi$ -field symmetry

$$\Sigma \rightarrow \Sigma + \epsilon_{\Phi}, \quad \Phi \rightarrow \Phi. \quad (10)$$

Thus the theory consists entirely of  $(N + 2)$  free scalar fields and is exactly soluble. The most general homogeneous vacuum solution of equations of motion

$$\ddot{\Phi} = \ddot{\Sigma} = 0; \quad \vec{f} = \text{constant} \quad (11)$$

is given by

$$\begin{aligned} -\frac{1}{2\kappa}\Sigma &:= (\rho - \phi) = Q_{\Sigma}t + A, \\ \Phi &:= e^{-2\phi} = Q_{\Phi}t + B \end{aligned} \quad (12)$$

where  $Q_{\Sigma}, Q_{\Phi}, A, B$  are integration constants specified by initial conditions. In particular  $Q_{\Sigma}, Q_{\Phi}$  denote conserved charges associated with the two classical symmetries Eqs.(9), (10). Imposing the constraint Eq.(8)

$$\ddot{\Phi} + \frac{1}{\kappa}\dot{\Phi}\dot{\Sigma} = 2Q_{\Phi}Q_{\Sigma} = 0, \quad (13)$$

cosmological solutions are classified into two distinct branches depending on whether each conserved charge vanishes or not:  $Q_{\Sigma} = 0, Q_{\Phi} \neq 0$  or  $Q_{\Sigma} \neq 0, Q_{\Phi} = 0$ .

The first branch having  $Q_{\Sigma} = 0, Q_{\Phi} \neq 0$  may be expressed as

$$\rho = \phi + \log 2\tilde{M}; \quad e^{-2\phi} = -8\tilde{M}t \quad (14)$$

after suitable shift of dilaton and coordinate-time  $t$ . Reality condition for dilaton field  $\phi$  then restricts  $-\infty < t \leq 0$ . For this interval, by introducing comoving cosmic time  $\tau := -(-2\tilde{M}t)^{1/2}$  that ranges over  $-\infty < \tau \leq 0$ , the spacetime metric may be expressed as

$$(ds)^2 = -\frac{\tilde{M}^2}{-2\tilde{M}t}[dt^2 - dx^2] = -[d\tau^2 - (\frac{\tilde{M}}{-\tau})^2 dx^2]. \quad (15)$$

The comoving scale factor  $a(\tau) = \tilde{M}/(-\tau)$  shows that this branch describes super-inflationary evolution. Dilaton in this branch evolves as  $\phi = -\log(-2\tau)$ .

The second  $Q_\Sigma \neq 0, Q_\Phi = 0$  branch is similarly expressed as

$$\rho = \phi + Mt; \quad e^{-2\phi} = M^{-2}. \quad (16)$$

The spacetime metric describes an expanding universe

$$(ds)^2 = -M^2 e^{2Mt}[dt^2 - dx^2] = -[d\tau^2 - (M\tau)^2 dx^2] \quad (17)$$

where comoving time  $\tau := \exp Mt$  ranges over  $0 \leq \tau < \infty$ . Universe expanding linearly in time is known as Milné universe. The Milné universe is a flat two-dimensional counterpart of the FRW-type universe. Dilaton in Milné universe branch is frozen to a constant value.

It is straightforward to recognize that the above two branches are mapped each other by scale-factor duality as in higher dimensions. In fact the two branches were known as anisotropic universes of Bianchi I type [8]. Is it then possible to complete the super-inflation and reheat to the Milné universe by matching the two branches at  $\tau = 0$ ? That this is not possible is immediately recognized from singular behavior of curvature scalar and string coupling constant in the super-inflation branch

$$g_{\text{st}} = \frac{1}{-2\tau} \rightarrow \infty, \quad R = (\frac{2}{-\tau})^2 \rightarrow \infty \quad (18)$$

in contrast to regular  $g_{\text{st}} = \text{finite}$ ,  $R = 0$  in the Milné branch. Thus any cosmological vacua belonging to the super-inflation branch end up with infinite curvature and string coupling and cannot escape out of that branch forever. Nature of the difficulty is inherently the same as in higher dimensional string inflationary cosmology, hence, may be viewed as two-dimensional version of the aforementioned *graceful exit problem*.

On a closer look, however, it is clear that one has to go beyond classical description to string cosmology. Since curvature  $R$  and string coupling  $g_{\text{st}}$  in the super-inflationary branch diverge as  $\tau \rightarrow 0^-$ , quantum corrections cannot be neglected. It may even be that the quantum back reactions are sufficiently strong enough to wash out the classical divergences Eq.(18) that a smooth transition to the Milné branch becomes possible. One-loop quantum correction is provided by conformal anomaly of  $N$ -component matter, reparametrization ghosts, dilaton and conformal modes. After adding local, covariant counterterms to retain the two classical symmetries Eqs.(9),(10) for exact solvability [6,7], the quantum effective action is given by

$$S_{\text{eff}} = S_0 - \frac{\kappa}{2} \int \frac{d^2x}{2\pi} \sqrt{-g} [R \frac{1}{\square} R + 2\phi R] \quad (19)$$

where  $\kappa = (N - 24)/24$ . The second term in Eq.(19) is a local, covariant counterterm and is added to retain the two classical symmetries Eqs.(9),(10), hence, the exact solvability. This is most easily seen from the fact that the effective action  $S_{\text{eff}}$  has exactly the same form as the classical one Eq.(7) in terms of quantum corrected fields

$$\Sigma = 2\kappa(\phi - \rho); \quad \Phi = e^{-2\phi} + \kappa\rho. \quad (20)$$

Structure of the kinetic term then suggests that quantum theory defined in terms of  $S_{\text{eff}}$  introduces a new expansion parameter [9]

$$g_{\text{eff}}^2 = 1/|2e^{-2\phi} - \kappa|. \quad (21)$$

The theory in fact corresponds to an exactly soluble conformal field theory [6]. The constraint equations Eq.(8) are also modified to

$$\partial_{\pm}^2 \Phi + \frac{1}{\kappa} \partial_{\pm} \Phi \partial_{\pm} \Sigma + (\partial_{\pm} \vec{f})^2 - \kappa \left( \frac{1}{2} \partial_{\pm} \Sigma + t_{\pm}(x^{\pm}) \right) = 0. \quad (22)$$

The first integrals  $t_{\pm}(x^{\pm})$  come from nonlocality of conformal anomaly and are determined by boundary conditions. We set  $t_{\pm} = 0$  by demanding the correct classical behavior for flat Minkowski spacetime vacuum with a constant string coupling.

Since the equations of motion are the same as classical ones Eq.(11), the most general homogeneous *quantum* solutions are again given by

$$\begin{aligned}\frac{1}{2|\kappa|}\bar{\psi} &:= (\rho - \phi) = Q_\Sigma t + A \\ \Phi &:= e^{-2\phi} - |\kappa|\rho = Q_\Phi t + B.\end{aligned}\tag{23}$$

For reasons that will become clear immediately, we have restricted  $\kappa < 0$ , viz,  $N < 24$ . We also note that the kinetic term of  $(\rho, \phi)$  is non-degenerate only for this choice. Moreover the constraint equation Eq.(22)

$$\begin{aligned}\kappa t_\pm &= 0 \\ &= \ddot{\Phi} + \frac{|\kappa|}{2}\ddot{\Sigma} - \frac{1}{|\kappa|}\dot{\Phi}\dot{\Sigma} = Q_\Phi Q_\Sigma,\end{aligned}\tag{24}$$

implies that there are again two solution branches. We now show that the two branches are not distinct but one and the identical, in sharp contrast to the classical situation. Both branches describe a universe evolving from super-inflation phase to Milné universe phase without encountering physical singularities.

The first quantum branch that corresponds classically to the super-inflationary branch is given by

$$\rho = \phi + \log 2\tilde{M}; \quad e^{-2\phi} - |\kappa|\rho = -8\tilde{M}t.\tag{25}$$

Combining the two, we have

$$\begin{aligned}e^{-2\rho} - \frac{|\kappa|}{4\tilde{M}^2}\rho &= -\frac{2t}{\tilde{M}} \\ e^{-2\phi} - |\kappa|\phi &= -8\tilde{M}t + |\kappa|\log 2\tilde{M}.\end{aligned}\tag{26}$$

To appreciate significance of  $\kappa < 0$  choice we note that the left hand sides of Eq.(26) are monotonic functions ranging over  $(-\infty, +\infty)$ . It also implies that real-valued  $\rho, \phi$  evolution is possible for  $-\infty < t < +\infty$ , hence, extends beyond the classical evolution range  $-\infty < t \leq 0$ . Since  $\phi$  is a monotonic function of  $t$ , one may take string coupling  $g_{\text{st}} = e^\phi$  as a convenient built-in clock.

At asymptotic past infinity  $t \approx -\infty$ ,

$$\begin{aligned}(ds)^2 &\rightarrow -\left(\frac{\tilde{M}^2}{-2\tilde{M}t}\right)[dt^2 - dx^2] = -[d\tau^2 - \left(\frac{\tilde{M}}{-\tau}\right)^2 dx^2], \\ \phi &\rightarrow -\log(-2\tau),\end{aligned}\tag{27}$$

hence, the universe starts with classical super-inflationary phase. It is straightforward to recognize that accelerating expansion  $\dot{a}(\tau) > 0$  and  $\ddot{a}(\tau) > 0$  is maintained for  $\tau \ll 0$ . On the other hand, at  $t \rightarrow +\infty$ ,

$$\begin{aligned}(ds)^2 &\rightarrow -e^{16\tilde{M}t/|\kappa|}[dt^2 - dx^2] = -[d\tau^2 - \left(\frac{8\tilde{M}}{|\kappa|}\tau\right)^2 dx^2], \\ \phi &\rightarrow \log \tau,\end{aligned}\tag{28}$$

where  $\tau \approx (|\kappa|/8\tilde{M}) \exp(8\tilde{M}t/|\kappa|)$ , thus behaves as Milné universe with a linear dilaton. Again it is straightforward to recognize that  $\dot{a}(\tau) > 0$  but  $\ddot{a}(\tau) < 0$ , viz, decelerating expansion for  $\tau \gg 0$ .

Given that  $\rho, \phi$  evolves continuously it is clear that initially super-inflationary accelerating expansion is changed into decelerating expansion at late time and approaches asymptotically to Milné universe. More insight to this may be gained from the behavior of scalar curvature as a function of coupling parameters  $g_{\text{st}}$  or  $g_{\text{eff}}$ :

$$\begin{aligned}R &: = 2e^{-2\rho}(d^2\rho/dt^2) \\ &= 16\frac{g_{\text{st}}^2}{(1 + |\kappa|g_{\text{st}}^2/2)^3} \\ &= 32g_{\text{eff}}^2(1 - |\kappa|g_{\text{eff}}^2)^2.\end{aligned}\tag{29}$$

It shows that the curvature vanishes at past/future infinity  $g_{\text{st}} \rightarrow 0/\infty$  and attain the maximum near ‘transition epoch’  $\tau \approx 0$  where  $g_{\text{st}} \approx 1$ . The last line shows that  $g_{\text{eff}}$  reaches the maximum value  $\sim 1/|\kappa|$  at the transition epoch. We conclude that the classical super-inflation branch has made successful ‘graceful exit’ to the Milné branch with the aid of quantum back reaction.

Similarly, the second quantum branch that corresponds classically to the Milné universe becomes



$$\rho = \phi + Mt; \quad e^{-2\phi} - |\kappa|\rho = M^{-2}, \quad (30)$$

hence,

$$\begin{aligned} e^{-2\rho} - |\kappa|e^{-2Mt}\rho &= M^{-2}e^{-2Mt} \\ e^{-2\phi} - |\kappa|\phi &= |\kappa|Mt + M^{-2}. \end{aligned} \quad (31)$$

At  $t \rightarrow -\infty$ , the branch behaves

$$\begin{aligned} (ds)^2 &\rightarrow -\exp\left(2(M^{-2} - e^{2Mt})/|\kappa|\right)[dt^2 - dx^2] \\ &= -[d\tau^2 - a(\tau)^2 dx^2]; \quad a(\tau) \approx \exp(e^{2M\tau}), \\ \phi &\rightarrow -M\tau, \end{aligned} \quad (32)$$

where  $\tau \approx \int^t dt' \exp(-e^{2Mt'}/|\kappa|) \rightarrow -\infty$ . It is straightforward to see that initially Minkowski flat universe with a linear dilaton enters quickly into inflationary branch  $\dot{a}(\tau) > 0$  and  $\ddot{a}(\tau) > 0$ . At  $t \rightarrow +\infty$ ,

$$\begin{aligned} (ds)^2 &\rightarrow -\frac{1}{|\kappa|Mt}e^{2Mt}[dt^2 - dx^2] \\ &= -[d\tau^2 - a(\tau)^2 dx^2]; \quad a(\tau) \approx \frac{\tau}{(|\kappa|\log \tau)^{1/2}}, \\ \phi &\rightarrow -\frac{1}{2}\log(\log \tau), \end{aligned} \quad (33)$$

hence, the universe exhibits decelerating expansion evolution  $\dot{a}(\tau) > 0, \ddot{a}(\tau) < 0$ . Asymptotically at  $\tau \rightarrow +\infty$ , the universe approaches a Milné universe with approximately constant dilaton. Scalar curvature expressed as a function of  $g_{\text{st}}, g_{\text{eff}}$  is given by

$$\begin{aligned} R &= C^2 \exp\left(-\frac{2}{|\kappa|} \frac{1}{g_{\text{st}}^2}\right) \frac{g_{\text{st}}^4}{(1 + |\kappa|g_{\text{st}}^2/2)^3} \\ &= (2C)^2 \exp\left(-\frac{1}{|\kappa|g_{\text{eff}}^2} + 1\right) g_{\text{eff}}^4 (1 - |\kappa|g_{\text{eff}}^2) \end{aligned} \quad (34)$$

where  $C := |\kappa|M \exp(1/|\kappa|M)$ . The  $g_{\text{st}}$  decreases monotonically from strong to weak coupling with  $\tau$ . The curvature scalar vanishes at asymptotic past and future infinities  $g_{\text{st}} \rightarrow 0, \infty$  but approaches a positive maximum value near transition epoch  $\tau \approx 0$ ,

$g_{\text{st}} = \mathcal{O}(1)$ . In terms of effective coupling the curvature vanishes at  $\tau \rightarrow \pm\infty$  where  $g_{\text{eff}} \rightarrow 0$  but becomes strong  $g_{\text{eff}} \rightarrow \mathcal{O}(1/|\kappa|)$  near transition epoch. It is interesting to note that the curvature scalar depends on both couplings in a suggestive form of nonperturbative effect associated with particle pair production. We conclude that the second quantum branch also shows *graceful exit* evolution of inflationary branch into Milné universe branch much the same manner as the first quantum branch.

One common aspect of both quantum branches is that the string coupling  $g_{\text{st}} = e^\phi$  evolves monotonically: either from weak to strong for the first branch or from strong to weak coupling for the second. Normally this indicates breakdown of string perturbation theory. On the other hand recent advent of string-string dualities [10] may suggest alternative interpretation: branch interpolation between strong and weak string coupling is equivalent to an interpolation between two weakly coupled phases of dual string pairs [11] such as type-I and heterotic and type-II/M-theory and heterotic strings. String duality then allows that  $g_{\text{st}} \rightarrow 0$  both at asymptotic past and future infinity but for different perturbative string theories. We also note that the effective coupling  $g_{\text{eff}}$  behaved in a similar way as the string-duality-applied  $g_{\text{st}}$ . Interestingly the transition epoch is precisely where both  $g_{\text{eff}}$  and string-duality-applied  $g_{\text{st}}$  is of order unity, hence, perturbation theory in all means should break down or the least well-behaved. Evidently this is where quantum back reaction is most pronounced.

In summary we have investigated effects of quantum back reaction in string cosmology. In exactly solvable two-dimensional model we have found that the effect is profound: classically disjoint super-inflation and Milné branches have become smoothly connected at quantum level. The consequence is that initially super-inflation phase with accelerating expansion gets retarded enough that exits gracefully to Milné universe with decelerating expansion. Thus a successful string inflationary cosmology is realized. Whether a similar mechanism is possible for higher dimensions is an interesting question and is under current investigation.

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